I ran through Einstein's second transformed equation in II.6 and got **almost** what he had, save that I swapped zeta and xi somewhere, as you will see. Either I made a mistake or Einstein did (give you one guess which is more likely!). Anyhow here it is.
To simplify it a little I set c=1 and dropped all the c's out, so we have

$$(1)\quad \begin{aligned} t &= \beta(\tau + v\xi) \\ x &= \beta(\xi + v\tau) \\ \tau &= \beta(t - vx) \\ \xi &= \beta(x - vt) \end{aligned}$$

and, of course,

$$(2)\quad \begin{aligned} \eta &= y \\ \zeta &= z \end{aligned}$$

The equation we're transforming is

$$(3)\quad \frac{\partial Y}{\partial t} = \frac{\partial L}{\partial z} - \frac{\partial N}{\partial x}$$

Applying (1) to $\frac{\partial Y}{\partial \tau}$ we have

$$(4)\quad \frac{\partial Y}{\partial \tau} = \beta\left[\frac{\partial Y}{\partial t} + v\frac{\partial Y}{\partial x}\right]$$

Plugging (3) into (4) we obtain:

$$(5)\quad \frac{\partial Y}{\partial \tau} = \beta\left[\frac{\partial L}{\partial z} - \frac{\partial N}{\partial x} + v\frac{\partial Y}{\partial x}\right]$$

Now we rewrite the partial of N by x in the Greek coordinates using the chain rule, and we rewrite the partial of Y by x in the Greek coordinates using the chain rule:

$$(6)\quad \begin{aligned} \frac{\partial N}{\partial x} &= \beta\left[-v\frac{\partial N}{\partial \tau} + \frac{\partial N}{\partial \xi}\right] \\ \frac{\partial Y}{\partial x} &= \beta\left[-v\frac{\partial Y}{\partial \tau} + \frac{\partial Y}{\partial \xi}\right] \end{aligned}$$

And we plug (6) into (5) to obtain:

$$(7)\quad \frac{\partial Y}{\partial \tau} = \beta\left[\frac{\partial L}{\partial z} + v\beta\frac{\partial N}{\partial \tau} - \beta\frac{\partial N}{\partial \xi} - v^2\beta\frac{\partial Y}{\partial \tau} + v\beta\frac{\partial Y}{\partial \xi}\right]$$

Next we multiply through by 1/beta, and then collect all the @Y/@tau and @N/@tau terms and move them to the left:

$$(8)\quad \left(\frac{1}{\beta}+v^2\beta\right)\frac{\partial Y}{\partial\tau}-v\beta\frac{\partial N}{\partial\tau} = \frac{\partial L}{\partial z}-\beta\frac{\partial N}{\partial\xi}+v\beta\frac{\partial Y}{\partial\xi}$$

With a little fiddling we realize that

$$(9)\quad \frac{1}{\beta}+v^2\beta = \beta$$

so, plugging (9) into (8) and rearranging a little, and replacing **z** with zeta, we obtain

$$\beta\frac{\partial}{\partial\tau}(Y-vN) = \frac{\partial L}{\partial\zeta}-\beta\frac{\partial}{\partial\xi}(N-vY)$$

which looks great, except that we have somehow contrived to swap zeta and xi. Either there is another step which allows you to actually swap them, or I made a mistake. I didn't spot the error in the course of transcribing it from paper but that certainly doesn't prove it's not there. :-(

Anyhow this is as far as I'm going on this; it's taking far too much time! I hope it was of some value.